International Finance

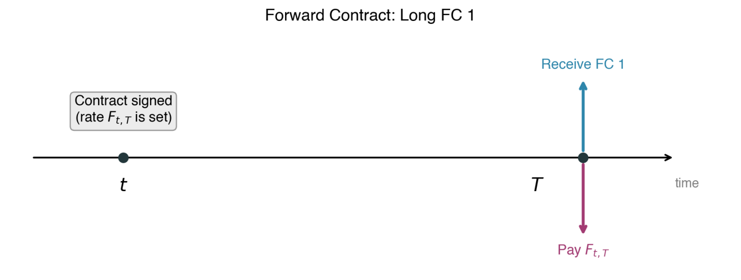
The Forward Market for Foreign Exchange

Main issues

- Understanding forward exchange contracts
- Hedging with forward contracts
- Covered interest rate parity (CIP)
- Valuation of forward contracts
- The cross-currency basis: when CIP “fails”

Definition of the forward exchange rate

The **forward exchange rate** $F_{t,T}$ is the price agreed upon today (t) at which one currency can be exchanged for another at a future date (T).



- Rate is set today — no uncertainty about the price
- No cash flows at initiation; exchange happens only at T

Forward quotes

FX Spot and CIP-Implied Forward Rates
Spot: 20 Feb 2026 | Rates: Jan 2026

	Spot	3M Forward	6M Forward	12M Forward
USD/EUR (HC per FC)	1.1781	1.1828	1.1874	1.1966
USD/GBP (HC per FC)	1.3500	1.3497	1.3495	1.3490
USD/JPY (HC per FC)	0.006452	0.006492	0.006533	0.006612

CIP-implied from spot rates and 3M interbank interest rates. Source: FRED.

Hedging with forward contracts

- **FC inflows** at a future date $\longrightarrow$ sell FC forward
- **FC outflows** at a future date $\longrightarrow$ buy FC forward

Hedging translates:

- A future FC cashflow whose HC value depends on the random $\tilde{S}_T$
- Into a known HC amount equal to $F_{t,T}$

The forward contract eliminates exchange rate uncertainty.

Example: hedging a foreign currency outflow

Suppose:

- You live in the US
- You are planning a vacation to the UK in 12 months
- Your budget is GBP 1,000

Questions:

1. What is your exchange rate exposure?
2. How can you hedge using a forward contract?
3. What is the cost of hedging?

The CIP formula

In frictionless markets:

$$F_{t,T} = S_t \frac{1 + r_{t,T}}{1 + r^*_{t,T}}$$

where

- S_t is the spot rate (HC per FC)
- $r_{t,T}$ is the domestic (HC) interest rate
- $r^*_{t,T}$ is the foreign (FC) interest rate

Where does this formula come from? We prove it via a replication argument.

CIP: numerical example

Given:

S_0 (USD/GBP)	1.26
$r_{0,1}^{USD}$	4.3%
$r_{0,1}^{GBP}$	4.5%
T	1 year

CIP gives us:

$$F_{0,1} = 1.26 \times \frac{1.043}{1.045} = 1.2576$$

GBP is at a **forward discount**: $F < S$.

Why? Because $r^{GBP} > r^{USD}$.

CIP proof: the replication argument

We compare two strategies that both deliver **GBP 1 at time T** with zero cash flow today:

Strategy A: Buy GBP 1 forward.

Strategy B:

1. Borrow $\frac{S_t}{1+r^*_{t,T}}$ in USD at rate $r_{t,T}$ for period T
2. Convert to GBP $\frac{1}{1+r^*_{t,T}}$ at spot rate S_t
3. Invest this in GBP at rate $r^*_{t,T} \rightarrow$ delivers GBP 1 at T

CIP proof: the payoff table

Strategy	Cashflow at t	Cashflow at T
A: Forward	0	Pay $F_{t,T}$; Receive GBP 1
B: Replication	Borrow USD $\frac{S_t}{1+r^*}$	Pay USD $S_t \frac{1+r}{1+r^*}$
	Buy GBP $\frac{1}{1+r^*}$ spot	
	Invest GBP at r^*	Receive GBP 1
	Net: 0	

Both strategies: zero cost at t, deliver GBP 1 at T. **No arbitrage** $\Rightarrow$ same cost at T:

$$F_{t,T} = S_t \frac{1 + r_{t,T}}{1 + r^*_{t,T}}$$

Implications of CIP

A forward contract can be **replicated** using three markets:

1. The spot FX market
2. The domestic money market
3. The foreign money market

Practical consequence: If you cannot trade forwards (e.g., restricted currency), you can create a *synthetic forward* using spot and money markets.

Conversely: If you observe spot, forward, and two interest rates, CIP pins down the relationship among all four. Any deviation is an arbitrage opportunity.

Forward premium and discount

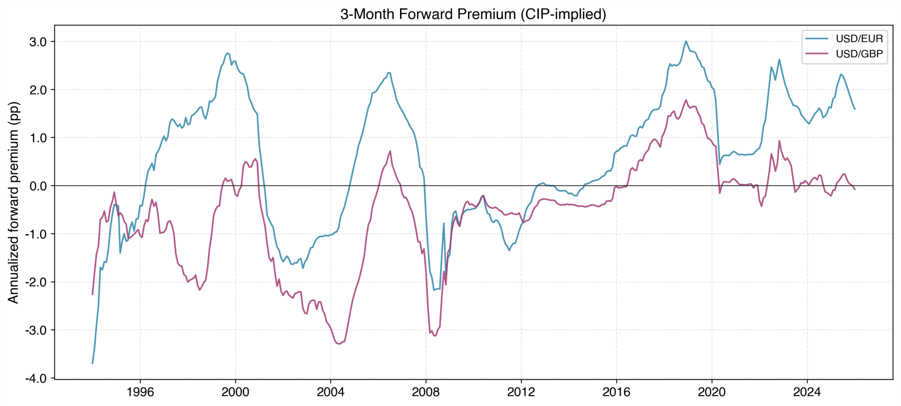
We can write CIP as:

$$\frac{F_{t,T}}{S_t} = \frac{1 + r_{t,T}}{1 + r^*_{t,T}}$$

- $F > S \Rightarrow$ FC is at a **forward premium** $\Rightarrow r > r^*$ (HC rate is higher)
- $F < S \Rightarrow$ FC is at a **forward discount** $\Rightarrow r < r^*$

Strong currencies offer low interest rates; weak currencies compensate with high rates.

Forward premium over time



Source: 3M interbank rates from OECD via FRED, 1994–2026.

CIP and corporate borrowing

The CIP can be rewritten as:

$$\underbrace{S_t^{-1}(1 + r^*_{t,T})F_{t,T}}_{\text{cost of borrowing HC 1 abroad}} = \underbrace{(1 + r_{t,T})}_{\text{cost of borrowing HC 1 at home}}$$

In frictionless markets, you cannot save money by borrowing in one currency rather than another (once you hedge the FX risk).

The low interest rate in JPY does not make JPY borrowing “cheap” — the forward premium offsets it exactly.

CIP: the mnemonic

In the ratio $\frac{1+r_{t,T}}{1+r^*_{t,T}}$, the **numerator interest rate** belongs to the **numerator currency** of S_t.

Example: If S_t is quoted as EUR/JPY:

$$F_{t,T} = S_t \frac{1 + r^{EUR}_{t,T}}{1 + r^{JPY}_{t,T}}$$

EUR is in the numerator of the exchange rate $\Rightarrow$ EUR rate goes in the numerator.

Valuing an existing forward contract

At $t+1$, value a forward initiated at t (maturity T, rate $F_{t,T}$) to buy FC 1.

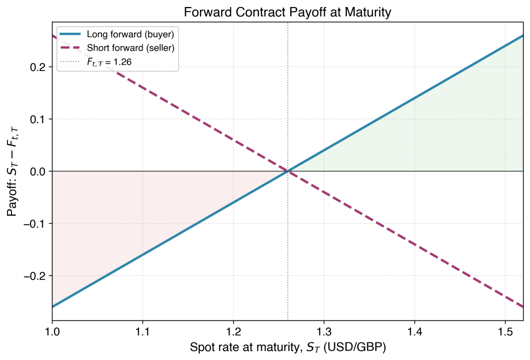
Cashflows at T: Receive FC 1, Pay HC $F_{t,T}$. **Present value at $t+1$:**

$$V_{t+1} = \frac{S_{t+1}}{1+r^*_{t+1,T}} - \frac{F_{t,T}}{1+r_{t+1,T}}$$

Using CIP: $\frac{S_{t+1}}{1+r^*_{t+1,T}} = \frac{F_{t+1,T}}{1+r_{t+1,T}}$, so:

$$V_{t+1} = \frac{F_{t+1,T} - F_{t,T}}{1+r_{t+1,T}}$$

Value at maturity



$$V(T) = S_T - F_{t,T}$$

Value at initiation

At the initiation date t:

$$V(t) = \frac{S_t}{1 + r^*_{t,T}} - \frac{F_{t,T}}{1 + r_{t,T}}$$

But CIP tells us $F_{t,T} = S_t \frac{1+r_{t,T}}{1+r^*_{t,T}}$, which means $\frac{S_t}{1+r^*_{t,T}} = \frac{F_{t,T}}{1+r_{t,T}}$.

Therefore: $V(t) = 0$.

The forward rate is chosen so the contract is fair at initiation — neither party pays the other.

Numerical example

3M forward for GBP at $F_{0,T}$ = USD/GBP 1.26. At initiation: $V_0 = 0$.

After 1 month, spot is S_{t+1} = 1.28:

$$V_{t+1} = \frac{1.28}{1 + r^*_{t+1,T}} - \frac{1.26}{1 + r_{t+1,T}} \approx 0.02 > 0 \quad \text{(positive value to buyer)}$$

At maturity, spot is S_T = 1.30:

$$V_T = 1.30 - 1.26 = 0.04 \text{ per GBP}$$

What is the market's certainty equivalent for $\tilde{S}_T$?

Suppose you are offered FC 1 at future date T. The present value can be computed two ways:

$$PV = \underbrace{\frac{E_t[\tilde{S}_T]}{1+\tilde{r}_{t,T}}}_{\text{risky CF at risk-adjusted rate}} = \underbrace{\frac{F_{t,T}}{1+r_{t,T}}}_{\text{safe CF at riskless rate}}$$

Therefore: $F_{t,T} = CEQ_t(\tilde{S}_T)$

The forward rate is the **certainty equivalent** of the future spot rate.

What does this mean?

- **CIP** links the *current* spot rate, the *current* forward rate, and interest rates.
- **CEQ** links the *future* spot rate and the *current* forward rate.

Important: $F_{t,T} = CEQ_t(\tilde{S}_T)$ does **not** imply $F_{t,T} = E_t[\tilde{S}_T]$.

Whether the forward rate equals the *expected* future spot rate is the question of the **Unbiased Expectations Hypothesis (UEH)**. This is tested separately and generally *fails* — which we will cover in a later lecture.

CIP deviations: the cross-currency basis

In theory: CIP holds exactly, and forward rates are pinned by interest rate differentials.

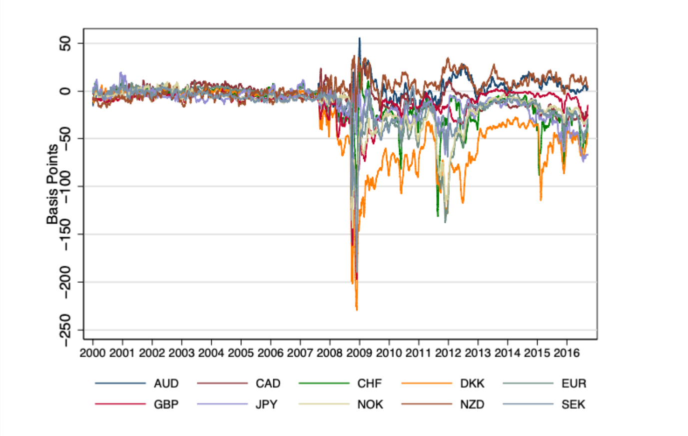
In practice: Small but persistent deviations exist, especially since 2008.

The **cross-currency basis** measures the deviation:

$$\text{basis} = \text{implied USD rate from FX swap} - \text{actual USD rate}$$

A negative basis means it is *more expensive* to obtain USD synthetically (via FX swaps) than to borrow USD directly.

CIP deviations: evidence from FX swap markets



Source: Du, Tepper and Verdelhan (2018), “Deviations from Covered Interest Rate Parity,” Figure 2.

Why the basis exists

- **Balance-sheet constraints:** Banks need capital to intermediate FX swaps. Basel III leverage ratios limit supply.
- **Funding stress:** Dollar shortage forces foreign banks to pay a premium for synthetic USD (GFC, COVID).
- **Regulatory costs:** Quarter-end window-dressing creates predictable spikes.
- **Segmentation:** Not all market participants can arbitrage freely (insurance companies, central banks).

For the firm: The basis is a risk signal. When it widens, hedging costs rise.

Summary

- **Forward contracts** allow hedging of future FC cashflows at a known rate.
- CIP: $F_{t,T} = S_t \frac{1+r}{1+r^*}$ – proved via replication (no arbitrage).
- **Forward valuation:** $V(t) = 0$ at initiation; $V(T) = S_T - F_{t,T}$ at maturity.
- **CIP implies** you cannot save money borrowing in another currency (after hedging).
- **Forward rate = certainty equivalent** of the future spot rate (but $\neq$ expected spot).
- **Cross-currency basis:** Real-world frictions create persistent CIP deviations. The basis matters for hedging cost and funding decisions.